

R-mode instability of strange stars and observations of neutron stars in LMXBs

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Using a realistic equation of state (EOS) of strange quark matter, namely, the modified bag model, and considering the constraints to the parameters of EOS by the observational mass limit of neutron stars, we study the r -mode instability window of strange stars, and find the same result as the brief study of Haskell, Degenaar and Ho in 2012 that these instability windows are not consistent with the spin frequency and temperature observations of neutron stars in LMXBs.

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I. INTRODUCTION

Ever since the realization in 1998 that r -modes, which are restored by the Coriolis force, are subjected to Chandrasekhar-Friedmann-Schutz (CFS) instability [1, 2] in a perfect fluid star with arbitrary rotation [3, 4], its study has received a lot of attention. It is easy to understand that for a realistic neutron star, the r -mode instability only happens in a range of spin frequencies and temperatures, the so-called r -mode instability window, which is decided by the competition between the gravitational-wave driven effect and viscous-dissipation damping effect to the modes [5]. Therefore, primarily the r -mode instability is an important physical mechanism that can prevent neutron stars from spinning up to its Kepler frequency (Ω_k , above which matter is ejected from the star's equator) [6–8], and gravitational waves emitted during the instability process could be detected [9–12]. In fact, some other aspects related to r -mode instability are also studied. For example, as an alternative explanation to the rapid cooling of neutron star in Cas A (which can be well explained by the superfluidity-triggering model [13–15]), it is suggested that the star experiences the recovery period following the r -mode heating process assuming the star is differentially rotating [16, 17].

Recently, as more and more temperature data of neutron stars in Low Mass X-ray Binaries (LMXBs) are presented through X-ray and UV observations [18, 19], many studies are trying to constrain the physics behind the r -mode instability of neutron stars, especially the equation of state (EOS) of cold dense matter, by the comparison between the r -mode instability window and spin frequency and temperature observations in these systems [18, 20–22].

In this paper, we will investigate the case of strange stars in detail following the brief study by Haskell, Degenaar and Ho [18]. Different from their work and other former works about r -modes in strange stars (eg. [6, 8]), our study is based on a realistic EOS of strange quark matter, namely, the modified bag model [23–26]. We give the timescales related to r -modes numerically; and what's more, before our study of the r -mode instability window, we fix the parameter space of EOS so that it can match the mass limit of neutron stars, which is

given by determining the mass of the millisecond pulsar PSR J1614-2230 to be $1.97 \pm 0.04 M_\odot$ in 2010 [27], and has been further updated by the measuring of the $2.01 \pm 0.04 M_\odot$ PSR J0348+0432 recently [28].

We only consider the case of bare strange stars in this work. Although strange stars can also support a thin crust of normal nuclear matter up to the neutron drip density [29], it only leads to minor changes to the maximum mass comparing with bare strange stars [30], and it also does not contribute significantly to the damping of r -modes [10, 18].

The plan of this paper is as follows. In Sec. II we briefly show the modified strange quark matter EOS taken by our study, and calculate the allowed parameter space following certain constraints. In Sec. III we give the inequality through which the r -mode instability window is determined, and the related gravitational-wave driven timescale and the viscous-damping timescales are also presented. In Sec. IV we compare the theoretical r -mode window with the spin frequency and temperature observations of neutron stars in LMXBs, and Sec. V is our conclusion and discussion.

II. EOS OF STRANGE QUARK MATTER AND CONSTRAINT BY THE MASS OF PSR J1614-2230

For strange quark matter, we take the modified bag model [23–26], in which up (u) and down (d) quarks are treated as massless particles while the strange quark (s) mass is a free parameter, and first-order perturbative corrections in the strong interaction coupling constant α_s are taken into account. The thermodynamic potential for the u , d and s quarks, and for the electrons are [25, 31]

$$\Omega_u = -\frac{\mu_u^4}{4\pi^2} \left(1 - \frac{2\alpha_s}{\pi}\right), \quad (1)$$

$$\Omega_d = -\frac{\mu_d^4}{4\pi^2} \left(1 - \frac{2\alpha_s}{\pi}\right), \quad (2)$$

$$\Omega_s = -\frac{1}{4\pi^2} \left\{ \mu_s \sqrt{\mu_s^2 - m_s^2} (\mu_s^2 - \frac{5}{2} m_s^2) \right.$$

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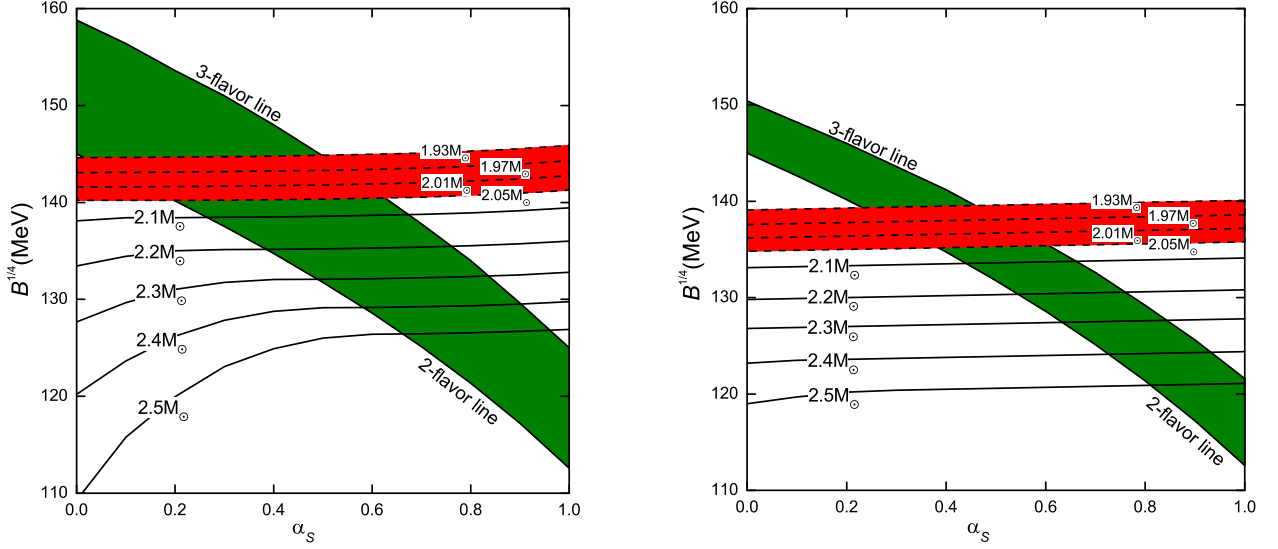


FIG. 1: The constraint to the parameters of the EOS of strange quark matter, namely, $B^{1/4}$ and α_s . The green shaded area corresponds to the allowed parameter space according to the constraints of the absolute stability of strange quark matter (3-flavor line) and the existence of nuclei (2-flavor line). The red shaded area marks the parameter space which have the maximum mass as PSR J1614-2230 ($M = 1.97 \pm 0.04 M_\odot$) and PSR J0348+0432 ($M = 2.01 \pm 0.04 M_\odot$). The combinations of $B^{1/4}$ and α_s which could lead to the strange star maximum mass as $M = 2.1 M_\odot$, $2.2 M_\odot$, $2.3 M_\odot$, $2.4 M_\odot$, $2.5 M_\odot$ are also presented, separately. The two graphs are for $m_s = 100$ MeV (left panel) and $m_s = 200$ MeV (right panel), respectively.

$$\begin{aligned}
 & + \frac{3}{2} m_s^4 f(u_s, m_s) - \frac{2\alpha_s}{\pi} \left[3(\mu_s \sqrt{\mu_s^2 - m_s^2} \right. \\
 & \left. - m_s^2 f(u_s, m_s))^2 - 2(\mu_s^2 - m_s^2)^2 - 3m_s^4 \ln^2 \frac{m_s}{\mu_s} \right. \\
 & \left. + 6 \ln \frac{\sigma}{\mu_s} \left(\mu_s m_s^2 \sqrt{\mu_s^2 - m_s^2} - m_s^4 f(u_s, m_s) \right) \right] \Bigg\}, \quad (3)
 \end{aligned}$$

$$\Omega_e = -\frac{\mu_e^4}{12\pi^2}, \quad (4)$$

where $f(u_s, m_s) \equiv \ln((\mu_s + \sqrt{\mu_s^2 - m_s^2})/m_s)$, σ is a renormalization constant whose value is of the order of the chemical potentials [23], and we take $\sigma = 300$ MeV in this paper (note, there is a typo in Ref. [31] before the term $3m_s^4 \ln^2 \frac{m_s}{\mu_s}$ of Ω_s , it should be “-” as given by Ref. [25]).

Before the discussion of the r -mode instability window of strange stars and compare it with observations of neutron stars in LMXBs, we calculate the allowed parameter space of EOS of strange quark matter according to the following basic constraints [32–34]. First, the existence of quark stars composed of stable strange quark matter is based on the idea that the presence of strange quarks can lower the energy per baryon of the mixture of u , d and s quarks in beta equilibrium below the one of ^{56}Fe ($E/A \sim 930$ MeV) [35]. This constraint results in the “3-flavor line” in Fig. 1. The second constraint is given by the assuming that non-strange quark matter (two-flavor quark matter consists of only u and d quarks) in bulk has a binding energy per baryon higher than the one of the most stable atomic nucleus, ^{56}Fe , plus a 4 MeV correction coming from surface effects [23]. By imposing that

$E/A \geq 934$ MeV for non-strange quark matter, one ensures that atomic nuclei do not dissolve into their constituent quarks and gives the “2-flavor line” in Fig. 1. The last constraint is that the maximum mass must be greater than the masses of PSR J1614-2230 ($M = 1.97 \pm 0.04 M_\odot$) and PSR J0348+0432 ($M = 2.01 \pm 0.04 M_\odot$). This constraint can also be shown in Fig. 1, since for each set of parameters of the strange quark matter EOS (namely, m_s , $B^{1/4}$ and α_s), one can give a maximum mass by solving the Oppenheimer-Volkoff equations.

According to the above three constraints, the allowed parameter space can be decided in Fig. 1. The region between the “3-flavor line” and “2-flavor line” (the green shaded area) is allowed according to the first two constraints, but considering the third constraint, only a part of the green shaded area are allowed, namely, the part below the red shaded area. From Fig. 1, It could be found that for our EOS model, both for the cases of $m_s = 100$ MeV and $m_s = 200$ MeV, the constraint about the maximum mass results in $\alpha_s > 0$, which means the QCD corrections must be included, and it is the same result as given by Weissenborn *et al.* [33].

III. R-MODE INSTABILITY WINDOW

The r -mode instability window of a strange star is decided by the inequality

$$\frac{1}{\tau_{GW}} + \frac{1}{\tau_\eta} + \frac{1}{\tau_\zeta} < 0, \quad (5)$$

where τ_{GW} is the time scale of the growth of an r -mode due to the emission of gravitational waves, τ_η and τ_ζ are the dis-

sipation time scales due to shear viscosity and bulk viscosity, respectively. For a strange star with given spin frequency Ω and core temperature T which satisfy the above inequality, the r -mode in the star should increase exponentially, and the amplified r -mode will transmit angular momentum of the star to gravitational waves; therefore, the star should quickly leave the instability window, making vanishing small probability to observe it in that region in the $\Omega - T$ plane [19].

The growth time scale due to the emission of gravitational waves is given by Ref. [5]

$$\frac{1}{\tau_{GW}} = -\frac{32\pi G\Omega^{2l+2}}{c^{2l+3}} \times \frac{(l-1)^{2l}}{[(2l+1)!!]^2} \left(\frac{l+2}{l+1}\right)^{2l+2} \int_0^R \rho r^{2l+2} dr, \quad (6)$$

where Ω is the spin frequency of the star, ρ is the mass density in g cm^{-3} . In this paper, we only focus on the r -modes with quantum number $l = 2$ and azimuthal projection $m = 2$ because these are the dominant ones [5, 6].

The dissipation time scale due to shear viscosity is [5]

$$\frac{1}{\tau_\eta} = (l-1)(2l+1) \left[\int_0^R \rho r^{2l+2} dr \right]^{-1} \int_0^R \eta r^{2l} dr. \quad (7)$$

The shear viscosity of strange quark matter due to quark scattering was calculated in [36], and the results for $T \ll \mu$, where T is the temperature and μ is the quark chemical potential, can be presented as [6]

$$\eta \approx 1.7 \times 10^{18} \left(\frac{0.1}{\alpha_S} \right)^{5/3} \rho_{15}^{14/9} T_9^{-5/3} \text{g cm}^{-1} \text{s}^{-1}, \quad (8)$$

where $T_9 \equiv T/10^9 \text{ K}$, and $\rho_{15} \equiv \rho/10^{15} \text{ g cm}^{-3}$.

The dissipation time scale due to bulk viscosity is given by Refs. [21, 37, 38], considering the second order effects [39]

$$\frac{1}{\tau_\zeta} = \frac{4\pi}{690} \left(\frac{\Omega^2}{\pi G \bar{\rho}} \right)^2 R^{2l-2} \left[\int_0^R \rho r^{2l+2} dr \right]^{-1} \times \int_0^R \zeta \left(\frac{r}{R} \right)^6 \left[1 + 0.86 \left(\frac{r}{R} \right)^2 \right] r^2 dr, \quad (9)$$

where $\bar{\rho} \equiv M/(4\pi R^3/3)$ is the average density of the nonrotating star. The bulk viscosity of strange quark matter depends mainly on the rate of the non-leptonic weak interaction [40–42]

$$u + d \leftrightarrow s + u, \quad (10)$$

to good approximation the bulk viscosity is [42]

$$\zeta \approx \alpha T^2 / [\omega^2 + \beta T^4], \quad (11)$$

with α and β given by Ref. [42], and ω is the angular frequency of the perturbation. During the study of r -mode instability, ω is the angular frequency of the r -mode perturbation $\omega_r = 2m\Omega/(l+1)$, where Ω is the spin frequency of the star. For the dominant r -mode ($m = l = 2$), $\omega = \frac{2}{3}\Omega$. The low- T

limit ($T < 10^9 \text{ K}$) is enough for this work, and it turns out to be [8]

$$\zeta \approx 3.2 \times 10^{28} m_{100}^4 \rho_{15} T_9^2 \omega^{-2} \text{g cm}^{-1} \text{s}^{-1}, \quad (12)$$

where m_{100} is the strange quark mass in units of 100 MeV and all the other quantities are in cgs units.

IV. COMPARISON OF THE INSTABILITY WINDOW WITH OBSERVATIONS

By solving the inequality (5), together with Eqs. (6), (7) and (9) numerically for given parameter sets of EOS of strange quark matter, one can get the r -mode instability window of strange stars. Here, we want to stress that we will only discuss the parameter sets of strange quark matter EOS which reside in the allowed parameter space as shown in Sect. II.

Fig. 2 shows the r -mode instability window for strange stars with the canonical neutron star mass $M = 1.4 M_\odot$, and the observational data on the spin frequency and internal temperature of neutron stars in LMXBs, which is given by Gusakov, Chugunov, and Kantor [19]. The left panel is for $m_s = 100 \text{ MeV}$ and $B^{1/4} = 140 \text{ MeV}$, and the right panel is for $m_s = 200 \text{ MeV}$ and $B^{1/4} = 135 \text{ MeV}$ (For each given m_s , we select the nearly largest $B^{1/4}$ that allowed by the limit of observational neutron star mass according to Fig. 1, because it corresponds to smaller allowed α_S , which will lead to a smaller r -mode instability region as can be seen in Fig. 2). For the left panel, three curves are presented, which represent the case of $\alpha_S = 0.2$, $\alpha_S = 0.4$ and $\alpha_S = 0.6$, respectively; while for the right panel, we only shows two curves, namely, $\alpha_S = 0.4$ and $\alpha_S = 0.6$. The reason is that the parameter set $\alpha_S = 0.2$, $m_s = 200 \text{ MeV}$ and $B^{1/4} = 135 \text{ MeV}$ is not located in the allowed parameter space as discussed in Sect. II, more exactly, non-strange quark matter do not satisfy the condition $E/A \geq 934 \text{ MeV}$ for this parameter set. It could be seen that, although the case of $m_s = 200 \text{ MeV}$ and $B^{1/4} = 135 \text{ MeV}$ (right panel of Fig. 2) looks better, all the possible instability windows are not consistent with the spin frequency and temperature observations of neutron stars in LMXBs, which turns out to be the same conclusion as drawn by Haskell, Degenaar and Ho [18].

V. CONCLUSION AND DISCUSSION

Following the brief study by Haskell, Degenaar and Ho [18], we examine the instability window of strange stars in detail, and compare it with the spin frequency and temperature observations of neutron stars in LMXBs. Our work is based on a realistic EOS of strange quark matter, namely, the modified bag model. Besides the numerical calculation to the timescales related to r -modes, we also employ a delicate strategy, in which firstly, we calculate the allowed parameter spaces of EOS so that it can match the observed mass limit of neutron stars, and then the study of the instability window of strange star and its comparison with observations are carried out.

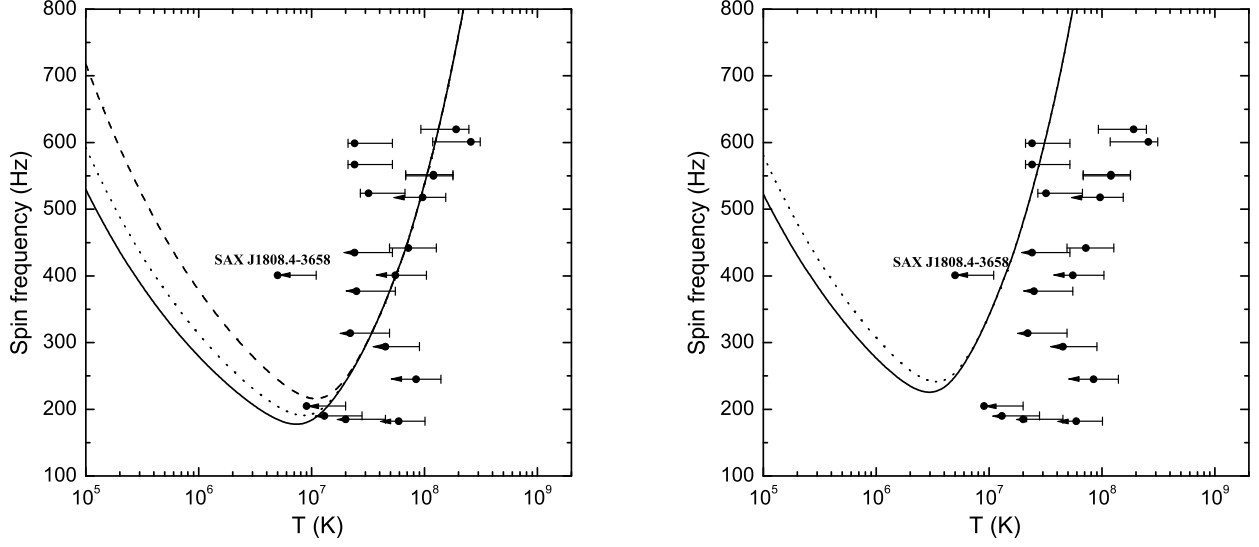


FIG. 2: R-mode instability window for strange stars with $M = 1.4 M_{\odot}$, comparing with the observational data on the spin frequency and internal temperature of neutron stars in LMXBs [19]. The left left panel is for $m_s = 100$ MeV and $B^{1/4} = 140$ MeV, and the right panel is for $m_s = 200$ MeV and $B^{1/4} = 135$ MeV. The dashed, dotted and solid curves correspond to $\alpha_S = 0.2$, $\alpha_S = 0.4$ and $\alpha_S = 0.6$, respectively (note, there is no dashed curve in the right panel because non-strange quark matter do not satisfy the condition $E/A \geq 934$ MeV for the parameter set $\alpha_S = 0.2$, $m_s = 200$ MeV and $B^{1/4} = 135$ MeV, which can be seen in Fig. 1).

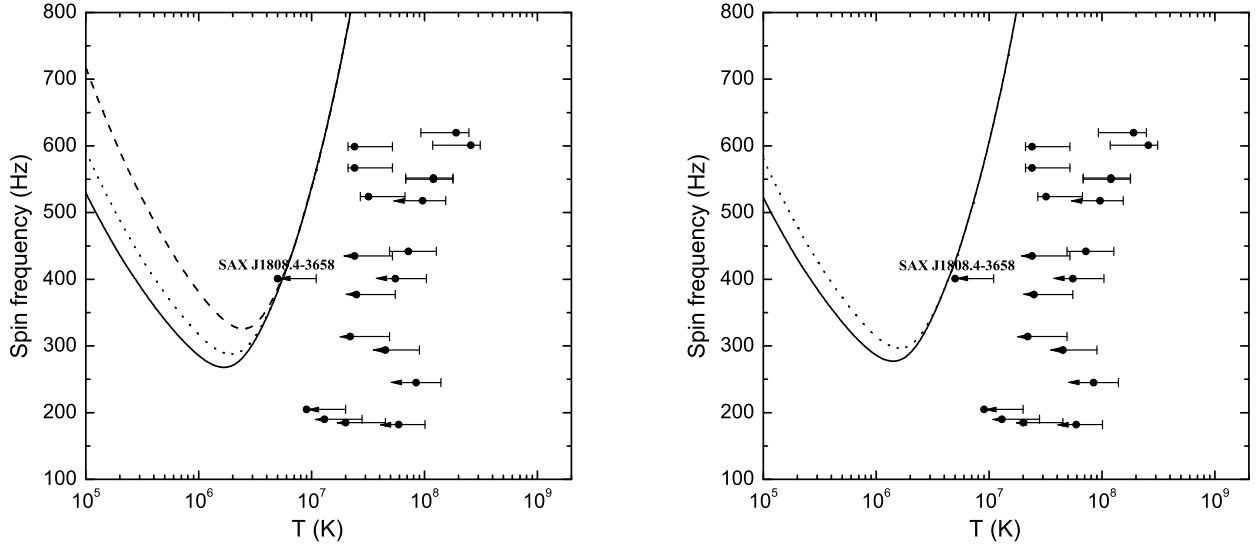


FIG. 3: Similar to Fig.1, but ζ is artificially taken as a 100 times larger one for the left panel and a 10 times larger one for the right panel.

Our study reiterates the conclusion given by Haskell, Degeaar and Ho [18] that all the possible instability windows are not consistent with the spin frequency and temperature observations of neutron stars in LMXBs. However, as far as the bulk viscosity of strange quark matter is considered in this paper, it is calculated under non-interacted Fermi liquid model [42]. If the interactions which lead to non-Fermi liquid effects is included, the bulk viscosity ζ can be increased by many orders of magnitude [43–46], and the instability window may be consistent with the observations. This possibility is shown

roughly in Fig. 3, using the same parameter sets of EOS as Fig. 2 but the bulk viscosity ζ is artificially taken as a 100 times larger one for the left panel and a 10 times larger one for the right panel. It could be seen from Fig. 3 that the instability window could almost be consistent with the observations under the above assumptions. The detailed study about these possibilities will be carried out in our future work.

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